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Stochastic Model of Production and Inventory Control using Dynamic Bayesian Network

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Abstract: Bayesian Network is a stochastic model, which shows the qualitative dependence between two or more random variables by the graph structure, and indicates the quantitative relations between individual variables by the conditional probability.

This paper deals with the production and inventory control using the dynamic Bayesian network. The probabilistic values of the amount of delivered goods and the production quantities are changed in the real environment, and then the total stock is also changed randomly. The probabilistic distribution of the total stock is calculated through the propagation of the probability on the Bayesian network. Moreover, an adjusting rule of the production quantities to maintain the probability of the lower bound and the upper bound of the total stock to certain values is shown.

Keywords: Graphical Modeling, Dynamic Bayesian Network, production inventory control, probability distribution

I. INTRODUCTION

The intellectual information processing systems to be constructed have to learn itself autonomously using the observed data of actual problem area. The Bayesian networks by now are widely accepted as powerful tools for representing and reasoning with uncertainty in decision-support systems[1][2].

This paper deals with the production and inventory control using the dynamic Bayesian network. The production quantities and delivered goods are changed randomly in this problem, and then the total stock is also changed randomly under these conditions. Firstly, the probabilistic distribution of the total stock is calculated through the propagation of the probability on the network. Furthermore, an adjusting rule of the production quantities to maintain the probability of the lower bound and upper bound of the total stock to a certain value is shown.

II. Bayesian Network

1. Bayesian Network

Bayesian Network is a probabilistic reasoning model that represents the dependency among random variables and gives a concise specification of joint probability distributions. This model is defined as a directed acyclic graph(DAG), which has conditional probabilities in each node. These nodes represent random variables in a domain and the directed links in the graph represent dependency between these nodes. The Bayesian network is a succession of nodes, to represent uncertain state variables, and linked by arrows which either represents causal or evidential relationships[6][7].

As examples, we consider variables x_i and x_j. When the variable x_j depends on the variable x_i, we note it as 「if $x_i=a$ then $x_j=b$」. Moreover, the dependence of the variable x_j on a lot of variables is written as 「if $x_1=a_1$, $x_2=a_2$, …, $x_t=a_t$ then $x_j=b$」. By the use of a conditional probability, the relation described above can be written as $P(x_j=b \mid x_j=a_1, x_2=a_2, \ldots x_t=a_t)$. These variables correspond to nodes, and the variables $A_j=\{x_1, x_2, \ldots, x_t\}$ are called as the parental nodes and the variables x_j is called as the child node in this case, then the conditional probability can be represented as $P(x_j \mid A_j)$.

A probability distribution can be calculated as

$$P(x_i)=\sum_{x_1}\cdots\sum_{x_{i-1}}\sum_{x_{i+1}}\cdots\sum_{x_n}P(x_1,\cdots,x_n) \qquad (1)$$

On the other hand, the joint probability distribution is represented based on the multiplication theorem as

$$P(x_1,\cdots,x_n)=P(x_1 \mid x_2,\cdots,x_n)P(x_2 \mid x_3,\cdots,x_n)\times \cdots \times P(x_{n-1} \mid x_n)P(x_n) \qquad (2)$$

For random variables $x_1, x_2, \ldots, x_n$, the joint probability distribution is represented as[8]

$$P(x_1 \cdots x_n)=\prod_j^n P(x_j \mid A_j,\theta_j) \qquad (3)$$

2. Dependence between nodes

In Bayesian network, the dependence between arbitrary nodes can be read directly from the graph. All conditional independence statements can be read a BN structure by using the important characteristics. As the characteristics, three connections on the Bayesian network are seial, convergence and divergence illustrated in Fig.1.

(1) serial

$P(A,B,C)=P(C|A,B)P(B|A)P(A)$

$$=P(C|B)P(B|A)P(A) \quad (4)$$

(2) convergence

$$P(A,B,C)=P(A|B,C)P(B,C)$$
$$=P(A|B,C)P(B)P(C) \quad (5)$$

(3) divergence

$$P(A,B,C)=P(B|A)P(C|A)P(A) \quad (6)$$

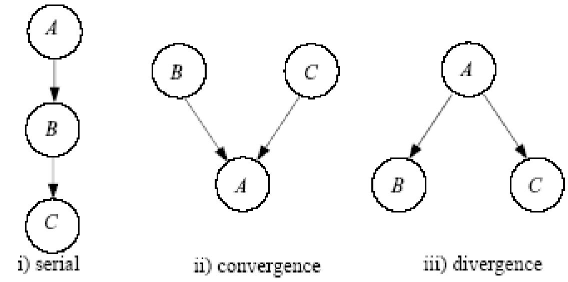

Fig.1 Connections on the Bayesian network

3. Dynamic Bayesian Network

Dynamic Bayesian Network(DBN) is an extended model of the Bayesian network to be applicable to dynamic systems, and the dynamic Bayesian network include the Kalman filter model and the hidden Markov model [9].

III. Stochastic Model of Production and Inventory Control

The production and inventory control using the Dynamic Bayesian Network is treated in this paper. The production quantities and delivery of goods are changed randomly in this problem, and then the total stock is also changed randomly under these conditions.

The production process is as that: n kind of products A^i_t ($i=1,2,\dots,n$) are manufactured and its forecast and an adjustment period of the production schedule hold between m days($t=1,2,\dots,m$, m : Forecast adjustment days). The production quantity of every day is $A^i_t \le A^i_{\max}$, and it is necessary to satisfy $\sum_i^n A^i_{\max} \le T^i_{\max}$. The daily production quantities are adjusted by the day's delivery of goods and stock the day before. However, when a large shipment is scheduled after the day, it is necessary to consider it. Moreover actual production quantities are assumed to be changed randomly according to unforeseen supply of materials, trouble of equipments, and states of labor environment. In addition, the delivery of goods of n days has been decided beforehand, however, it is also assumed that these are changed randomly according to the order for interrupt and outbreak cancels. Thus, the total stock of the product S^i_t of tth day can be expressed as

$$S^i_t = S^i_{t-1} + A^i_t - D^i_t \quad \dots\dots (7)$$

The stochastic model of the production and the inventory control considering the practical dependence of productions, deliveries, and inventories changed randomly is illustrated in Fig.2.

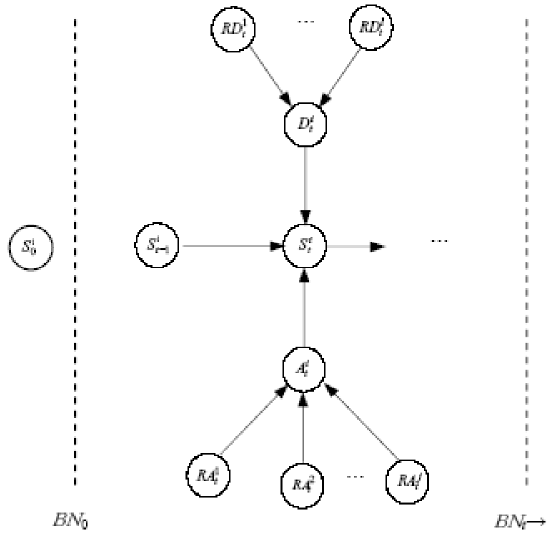

Fig.2 Stochastic model of production and inventory by Dynamic Bayesian Network

IV. Multi-production and Total stock amount maintenance production adjusting algorithm

1. Probabilistic Distribution of Total Stock

As an example, the production process is assumed to do for 30 days and five kinds of delivery of goods, production, and stock problems. The production quantity of every day is A^i_t=6000(i=1,2,3) and A^i_t=600(i=4,5), and it is necessary to satisfy $T^1_{\max}$18000(i=1,2,3) and $T^2_{\max}$1000(i=1,2,3).

The delivered goods and the production quantities of every day are scheduled beforehand as Table 1, and Table 2.

In the real environment, the delivered goods can be changed according to the order for interrupt and outbreak cancels. Therefore, the probability values for the amount of the delivered goods changed randomly is set as shown in Fig.3. Therefore, the conditional probability table of the delivered goods is given in Table 3.

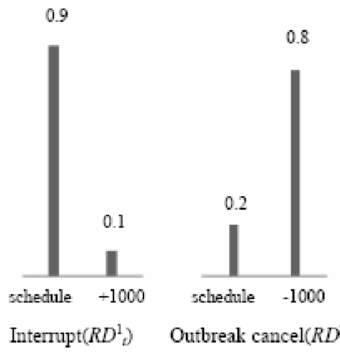

Fig.3 Prior probability of the delivered goods

Table 1. Schedule of the delivered goods

Day	1	2	3	4	5	6	7	8	9	10
Product D^1_t	3000	3000	4000	2000	10000	3000	3000	4000	2000	3000
Product D^2_t	4000	4000	3000	5000	3000	4000	4000	3000	3000	8000
Product D^3_t	3000	4000	5000	3000	5000	3000	4000	3000	3000	8000
Product D^4_t	300	400	200	200	300	300	300	400	200	700
Product D^5_t	500	300	300	400	400	300	100	800	200	300
Day	11	12	13	14	15	16	17	18	19	20
Product D^1_t	3000	5000	10000	3000	5000	3000	4000	4000	3000	3000
Product D^2_t	4000	4000	6000	3000	4000	8000	6000	4000	4000	4000
Product D^3_t	4000	4000	4000	3000	5000	5000	3000	4000	9000	2000
Product D^4_t	200	500	500	400	400	200	300	300	400	400
Product D^5_t	500	400	400	400	300	300	500	500	400	700
Day	21	22	23	24	25	26	27	28	29	30
Product D^1	4000	2000	10000	3000	5000	4000	2000	3000	3000	5000
Product D^2_t	4000	3000	3000	5000	3000	4000	4000	4000	8000	5000
Product D^3_t	6000	4000	3000	2000	4000	8000	3000	3000	8000	5000
Product D^4_t	300	500	500	900	200	300	300	300	300	400
Product D^5_t	500	400	400	400	400	400	300	800	300	300

Table 2. Schedule of the production quantities

Day	1	2	3	4	5	6	7	8	9	10
Product A^1_t	4000	3000	4000	5000	5000	5000	4000	5000	4000	4000
Product A^2_t	4000	3000	4000	4000	4000	4000	4000	3000	4000	4000
Product A^3_t	4000	6000	5000	5000	6000	5000	5000	5000	4000	6000
Product D^4_t	500	500	500	400	400	400	300	400	500	500
Product D^5_t	500	400	400	400	300	300	600	300	400	300
Day	11	12	13	14	15	16	17	18	19	20
Product A^1_t	4000	5000	4000	4000	4000	4000	3000	6000	4000	4000
Product A^2_t	4000	3000	5000	4000	3000	5000	6000	3000	4000	3000
Product A^3_t	4000	4000	5000	4000	4000	3000	5000	5000	5000	3000
Product D^4_t	400	500	500	400	400	400	500	300	400	400
Product D^5_t	500	400	400	400	300	300	500	500	400	300
Day	21	22	23	24	25	26	27	28	29	30
Product A^1_t	5000	4000	4000	4000	4000	4000	5000	4000	4000	4000
Product A^2_t	5000	3000	4000	4000	4000	4000	4000	3000	4000	5000
Product A^3_t	4000	6000	3000	5000	4000	4000	5000	5000	6000	5000
Product D^4_t	400	500	500	400	300	300	300	300	300	400
Product D^5_t	600	400	400	400	400	400	400	400	500	300

Table 3. Conditional Probability Table of the delivered goods

Conditional Probability Table		Interrupt		$P(RD^2_t)$
		schedule	+1000	
Outbreak	schedule	0.72	0.08	0.8
cancel	-1000	0.18	0.02	0.2
$P(RD^1_t)$		0.9	0.1	

In addition, the production quantity is also assumed that it can be changed randomly according to unforeseen supply of materials, trouble of equipments, and states of labor environment. Therefore, the probability values of the production quantities are set as shown in Fig.4. Therefore, the conditional probability table of the production quantities is given as in Table 4.

The probability distribution for the amount of the product A^i_t in stock of the ith day can be calculated from the next expression:

$$P(S^i_t) = \sum_{S^i_{t-1}} \sum_{A^i_t} \sum_{D^i_t} \sum_{RA^1_t} \sum_{RA^2_t} \sum_{RA^3_t} \sum_{RD^1_t} \sum_{RD^2_t} P(S^i_t, S^i_{t-1}, A^i_t, D^i_t, RA^1_t, RA^2_t, RA^3_t, RD^1_t, RD^2_t) \quad (9)$$

$$= \sum_{s_{t-1}^l \in \Omega S_{t-1}^l} \sum_{a_t^l \in \Omega A_t^l} \sum_{d_t^l \in \Omega D_t^l} \sum_{ra_t^1 \in \Omega RA_t^1} \sum_{ra_t^2 \in \Omega RA_t^2} \sum_{ra_t^3 \in \Omega RA_t^3} \sum_{rd_t^1 \in \Omega RD_t^1} \sum_{rd_t^2 \in \Omega RD_t^2} P(S_{t-1}^l = s_{t-1}^l, A_t^l = a_t^l, D_t^l = d_t^l, RA_t^1 = ra_t^1, RA_t^2 = ra_t^2, RA_t^3 = ra_t^2, RD_t^1 = rd_t^1, RD_t^2 = rd_t^2)$$

where,

$$P(S_t^l, S_{t-1}^l, A_t^l, D_t^l, RA_t^1, RA_t^2, RA_t^3, RD_t^1, RD_t^2) = P(S_t^l \mid S_{t-1}^l, A_t^l, D_t^l) \times P(A_t^l \mid RA_t^1, RA_t^2, RA_t^3) \times P(D_t^l \mid RD_t^1, RD_t^2) \quad (10)$$

The probability distribution of the total stock $S^1{}_t$, $S^2{}_t$, and $S^3{}_t$ of 10 days based on the initial production schedule of Table 2 and the schedule of the amount for delivery of goods of Table 1 are shown in Fig.5, Fig.6 and Fig.7.

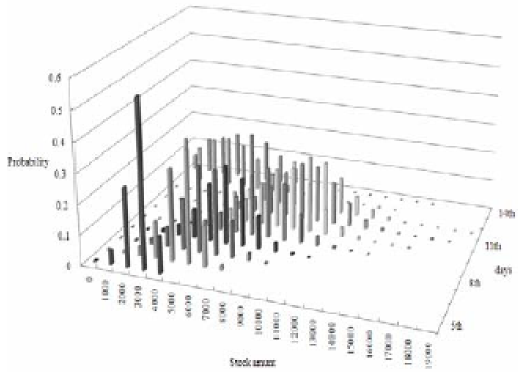

Fig.5 Probability distribution for the amount of the product $A^1{}_t$ in stock

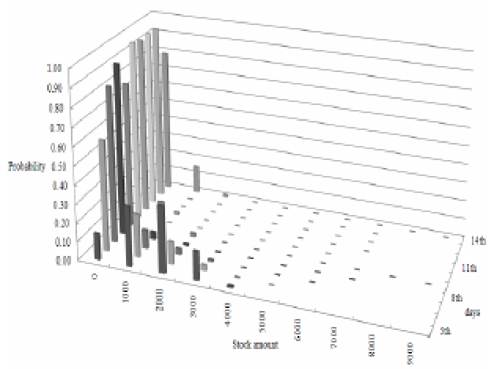

Fig.6 Probability distribution for the amount of the product $A^2{}_t$ in stock

2. Renewal of Conditional Probability

Here, the forecast adjustment day is 10 days, and the conditional probability is forecasted based on the production and the delivery of goods changed in randomly. The renewal of the conditional probability is shown Fig.8.

When the *4th* delivered goods and production quantities are determined the probability distribution of the amount of the forecast stock of product $A^1{}_t$ are shown in Fig.9 according to the updated conditional probability.

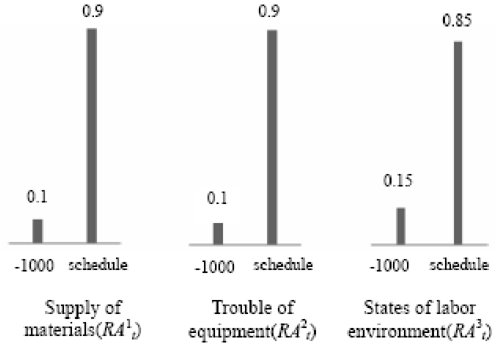

Fig.4 Prior probability of production quantities

Table 4. Conditional Probability Table of the production quantities

Conditional Probability Table		Supply of materials: schedule		Supply of materials: -1000		$P(RA^2{}_t)$
		States of labor environment: schedule	States of labor environment: -1000	States of labor environment: schedule	States of labor environment: -1000	
Trouble of equipment	schedule	0.6885	0.1215	0.0765	0.0135	0.9
	-1000	0.0765	0.0135	0.0085	0.0015	0.1
$P(RA^3{}_t)$		0.85	0.15	0.85	0.15	
$P(RA^1{}_t)$		0.9		0.1		

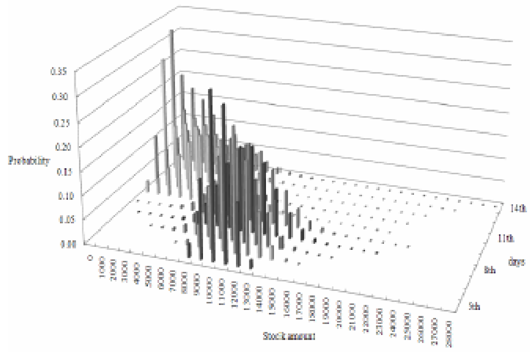

Fig.7 Probability distribution for the amount of the product A^3 in stock

The probability for the lack of delivery of the product $A^1{}_t$ is high in the 5th day, the 13th day, and the 14th day illustrated in Fig.9, because a large amount of delivery goods is scheduled to *5th* day and *13th* day as shown in Table 1.

The total stock is decided by the amount of stock the day before, the delivered goods and the production quantities on the day. But, it is necessary to consider a large amount of shipment after the day and to keep more than a certain amount. Then, an adjusting rule of the production schedule to maintain the probability of the lower limit and upper bound value of the total stock to a certain value is necessary.

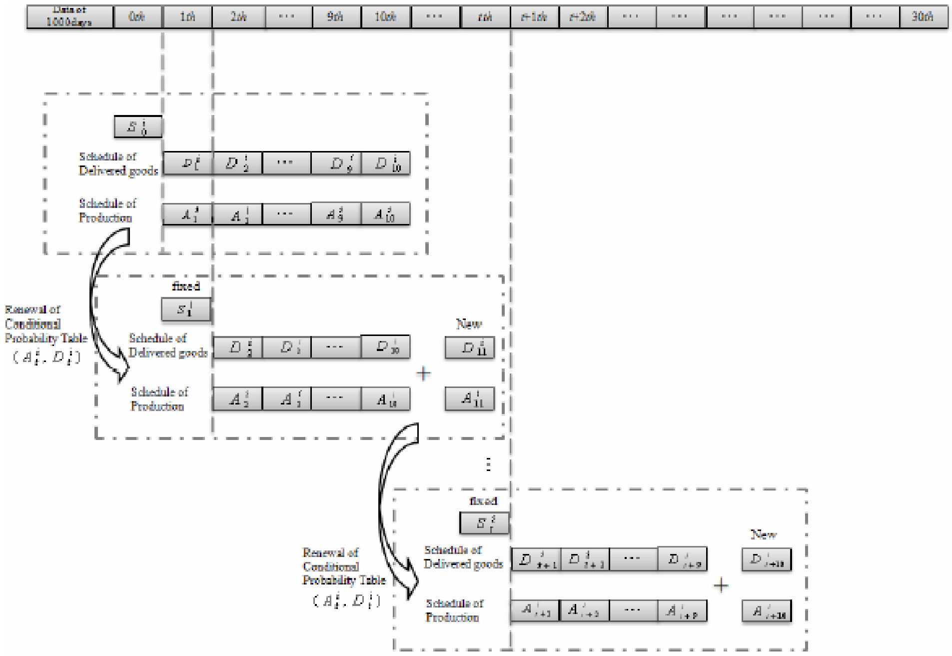

Fig.8 Renewal of conditional probability

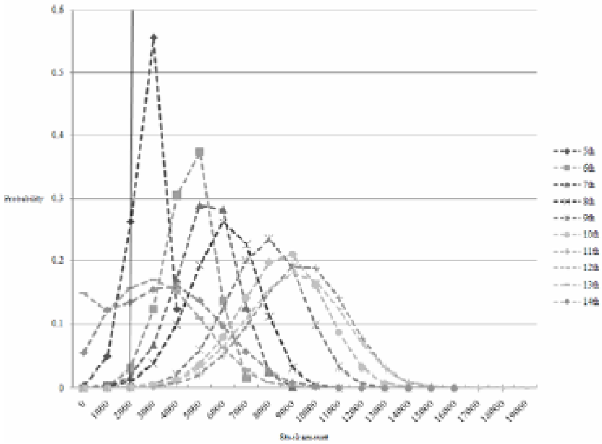

Fig.9 Probability distribution of the amount for the product A^1_t in stock

3. Production adjusting algorithm for proper stock amount maintenance

As an example, the production schedule is improved as that the probability of a lower limit or less of the amount for the total stock and the probability of a upper bound or more of it don't exceed 5%, respectively. Here the lower limit and the upper bound of the amount for the total stock A^1_t, A^2_t, and A^3_t are 2000 and 15000, and the lower limit and the upper bound of the amount for the total stock A^4_t and A^5_t are 200 and 1300, respectively.

The production schedule can be automatically updated by the proposed algorithm. When the 4*th* day is determined the production schedule updated by the adjusting rule is shown in Table 5.

A new probability distribution of the total stock by the production schedule corrected is as shown in Fig.10.

And, the probability for the amount of product A^1_t in stock to be 2000 or less and the probability for the amount of the product A^3_t in stock to be 15000 or more are shown in Table6 and Table7.

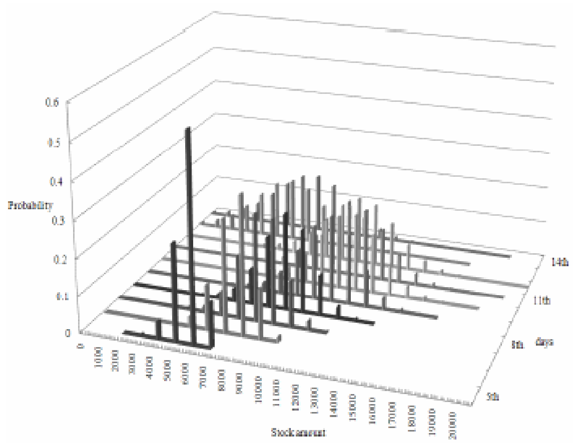

Fig.10 Probability distribution for the amount of the product A^1_t in stock by the adjusting rule

VI. CONCLUSION

This paper dealt with the production and inventory control using the dynamic Bayesian network. Under the situations in which the production quantities and delivered goods are changed randomly, the total stock is also changed randomly under these conditions.

Firstly, the probabilistic distribution of the total stock was calculated through the propagation of the probability on the network. Furthermore, the adjusting rule of the production quantities to maintain the probability of the

Table 5 The production schedule up dated by the adjusting rule

Day	5	6	7	8	9	10	11	12	13	14
Product A^1	6000	5000	4000	5000	4000	3800	3000	5200	6000	4000
Product A^2	5000	6000	4800	2000	3200	6000	6000	5800	6000	4000
Product A^3	5800	3000	4000	4000	3000	6000	6000	5000	5000	3000
Product A^4	300	300	300	400	300	600	300	500	500	500
Product A^5	400	400	280	600	400	300	520	400	500	400

Table 6 The probability for the amount of the produce A^1_t in stock to be 2000 or less

Day	5	6	7	8	9	10	11	12	13	14
Initial	31.56%	3.48%	2.16%	1.32%	0.18%	0.12%	0.07%	0.19%	42.42%	31.05%
Final	0%	0%	0%	0%	0%	0%	0%	0%	3.12%	1.94%

Table7 The probability for the amount of the produce A^3_t in stock to be 15000 or more

Day	7	8	9	10	11	12	13	14	15	16
Initial	14.67%	32.02%	23.86%	0.21%	0.05%	0.01%	0.028%	0.08%	0.01%	0%
Final	1.73%	3.25%	3.38%	0.03%	0.25%	0.86%	2.00%	3.73%	1.42%	1.19%

lower limit and upper bound of the total stock to a certain value was deduced.

As the result, the production schedule could be updated so as not to exceed the probability of the lower limit and upper bound of the amount of stock specified.

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